For the region after the reattachment site

$$\mathrm{Nu} = c\,(x/d_0^{\mathrm{h}})^{-1}\,\mathrm{Re}. \tag{3}$$

The curves of the excess temperatures in dimensionless coördinates in different cross sections of the initial (entrant) length of the jet merge into a single universal curve. The isotherms of the dimensionless temperatures for all the investigated types of chambers are straight radial lines converging at the pole of the jet in the core and at the nozzle orifice in the boundary layer.

The assumption of additivity of the radiant and convective heat-transfer components can be used to determine the total heat transfer [2] to the walls of a combustion chamber in the first approximation.

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SPECTRAL CHARACTERISTICS OF TURBULENCE IN ROTATING CHANNELS

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UDC 532.517.4

Spectral characteristics of turbulent flow in rotating channels are considered with allowance for the pulsating Coriolis forces and thermal heterogeneity of the flow.

As a fluid flows over a rotor of a turbomachine, the volume forces due to the rotation and curvature of the blade surfaces affect the character of the turbulent flow at the sides of condensation and rarefaction of the interblade channel.

Heterogeneity of the thermal and density fields leads to the appearance of Archimedian forces, which can intensify or reduce the turbulence. Similar flows in the atmosphere and ocean are often called flows with unstable and stable stratification. From now on, this terminology will also be used to characterize fluid flow in the rotating channel.

Let us consider the balance equation for the turbulent energy [1] of the flow of an incompressible fluid in a rotating radial channel. We direct the x axis along the channel surface, the y axis, in a perpendicular dirrection, into the flow, and we place the z axis on the axis of rotation:

$$\varepsilon = \langle u'_\alpha X'_\alpha \rangle - \langle u'_\alpha u'_\beta \rangle \frac{\partial u_\beta}{\partial x_\alpha} \mp \frac{2\omega u_\beta}{T} \alpha_T K \frac{\partial T}{\partial x_\alpha}, \tag{1}$$

where X'_α are the pulsations of the Coriolis forces, equal to $2\omega u'_\beta$. The upper sign designates the flow on the inlet side of the channel, while the lower sign designates the flow on the outlet side [2].

Writing the velocity correlation as

$$\langle u'v' \rangle = -K \frac{\partial u}{\partial y}, \tag{2}$$

we obtain

$$\varepsilon = K\left(\frac{\partial u}{\partial y}\right)^2 - 4\omega K \frac{\partial u}{\partial y} \mp \frac{2\omega u}{T} \alpha_T K \frac{\partial T}{\partial y}. \tag{3}$$

S. M. Kirov Kazan' Institute of Chemical Technology. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 50, No. 4, pp. 547-554, April, 1986. Original article submitted December 29, 1984.

0022-0841/86/5004-0382 $12.50

In order to obtain a spectral form of Eq. (3), we write the Kolmogorov equation for locally homogeneous turbulence [3, 4], taking the principal direction to be along the y axis:

$$E_3(k) = 2\nu \int_k^\infty k^2 E(k)\, dk + H(k), \tag{4}$$

where $E_3(k)$ is the amount of energy over the spectrum transferred per unit time from the large-scale perturbations with wave numbers less than k to other turbulent perturbations; E(k) is the spectrum of the kinetic energy; and the value of H(k) takes into account the energy delivered to the turbulent flow. For a stratified fluid in a gravitational field under the action of Archimedian forces, the value of H(k) is determined by the formula given in [3]

$$H(k) = \frac{g}{T} \int_k^\infty E_{Tv}(k)\, dk.$$

In the rotating channel, the Coriolis forces and heterogeneous pulsating thermal field, determined by the heterogeneous velocity field and by the transfer of heat from the flow to the walls, lead to the appearance of Archimedian forces, where the work done by these forces can be represented by the term $\frac{2\omega u}{T} \int_k^\infty E_{Tv}(k)\, dk$.

Pulsations of the Coriolis forces add to the changes in the turbulent energy of the flow, their contribution being defined as $4\omega \int_k^\infty E_{uv}(k) dk$, where $E_{uv}(k)$ is the spectrum of the turbulent flow of the momentum.

Taking into account the aforementioned, we write the spectral equation (4) in the form

$$E_3(k) \mp \frac{2\omega u}{T} \int_k^\infty E_{Tv}(k)\, dk - 4\omega \int_k^\infty E_{uv}(k)\, dk = 2\nu \int_k^\infty k^2 E(k)\, dk. \tag{5}$$

Let us represent the equation for the heat inflow in the form

$$\frac{\partial T}{\partial t} = v \left(\frac{\partial T}{\partial y} + \delta \right) = \chi \frac{\partial^2 T}{\partial y^2}. \tag{6}$$

We determine the adiabatic gradient of the temperature δ by differentiating the Bernoulli equation for the relative motion with respect to y

$$\delta = \left(\frac{\partial T}{\partial y} \right)_{\text{ad}} = -\frac{u}{c_p} \frac{\partial u}{\partial y}. \tag{7}$$

Let us note that the value of δ for large rotational velocities may exceed 100 deg/m.

Let us represent the equation of the heat inflow in the spectral form [5]

$$E_{uTT}(k) + \delta \int_k^\infty E_{Tv}(k)\, dk = \chi \int_k^\infty k^2 E_T(k)\, dk, \tag{8}$$

where $E_{uTT}(k)$ describes the transfer of the measure of thermal heterogeneity over the spectrum; $E_T(k)$ is the spectrum of the thermal field.

In accordance with the Monin–Heisenberg theory [3, 5], we write the following expressions for the spectra:

$$\begin{gathered} E_3(k) = K(k) \left[\left(\frac{\partial u}{\partial y} \right)^2 + 2 \int_{k_0}^k k^2 E(k)\, dk \right], \\ E_{uTT}(k) = \alpha_T K(k) \left[\left(\frac{\partial T}{\partial y} \right)^2 + \int_{k_0}^k k^2 E_T(k)\, dk \right], \\ \int_k^\infty E_{Tv}(k)\, dk = \pm \alpha_T K(k) \left[\left(\frac{\partial T}{\partial y} \right)^2 + \int_{k_0}^k k^2 E_T(k)\, dk \right]^{1/2} \end{gathered} \tag{9}$$

and for the spectral coefficient of the turbulent viscosity

$$K(k)=\gamma_0\left[\int_k^\infty k^{-2}E(k)\,dk\right]^{1/2}, \tag{9a}$$

where γ_0 is a numerical constant of order unity.

The spectral flow of the momentum in accordance with the proposition of Chen [6] and the restriction on the flow according to Obukhov can be written in the form

$$\int_k^\infty E_{uv}(k)\,dk=K(k)\left[\left(\frac{\partial u}{\partial y}\right)^2+2\int_{k_0}^k k^2E(k)\,dk\right]^{1/2}. \tag{10}$$

Taking into account Eqs. (9) and (10), the spectral equations for the balance of the turbulent energy and the intensity of the temperature pulsations (5) and (8) can be written as

$$\gamma_0\left[\int_k^\infty k^{-2}E(k)\,dk\right]^{1/2}\left\{\left(\frac{\partial u}{\partial y}\right)^2+2\int_{k_0}^k k^2E(k)\,dk-4\omega\left[\left(\frac{\partial u}{\partial y}\right)^2+\right.\right.$$
$$\left.\left.+2\int_{k_0}^k k^2E(k)\,dk\right]^{1/2}\mp\frac{2\omega u}{T}\alpha_T\left[\left(\frac{\partial T}{\partial y}\right)^2+\int_{k_0}^k k^2E_T(k)\,dk\right]^{1/2}\right\}=\varepsilon, \tag{11}$$

$$\alpha_T\gamma_0\left[\int_k^\infty k^{-2}E(k)\,dk\right]^{1/2}\left\{\left(\frac{\partial T}{\partial y}\right)^2+\int_{k_0}^k k^2E_T(k)\,dk+\delta\left[\left(\frac{\partial T}{\partial y}\right)^2+\int_{k_0}^k k^2E_T(k)\,dk\right]^{1/2}\right\}=\varepsilon_T, \tag{12}$$

where $\delta=\pm 2\omega u/c_p$.

In Eqs. (11) and (12) the integrals on the right sides of Eqs. (5) and (8) are replaced respectively by ε and ε_T [3].

When $k=k_0$ and $K(k)=K$, we obtain from Eqs. (11) and (12) the macroscopic balance equations for the turbulent energy (3) and a measure of the heterogeneity of the thermal field

$$\varepsilon_T=\alpha_T K\left(\frac{\partial T}{\partial y}\right)^2+\delta\alpha_T K\frac{\partial T}{\partial y}, \tag{13}$$

which allow us to determine ε and ε_T from the data on distributions of the mean velocity, temperature, and the coefficient of turbulent viscosity in the flow.

We will seek a solution to Eqs. (11) and (12), according to [5], in the form

$$E(k)=\left(\frac{\varepsilon}{\gamma_0}\right)^{2/3}L^{5/3}f(x),\quad E_T(k)=\frac{\varepsilon_T}{\alpha_T\gamma_0}\left(\frac{\varepsilon}{\gamma_0}\right)^{-1/3}L^{5/3}f_T(x), \tag{14}$$

assuming for the scale of buoyancy L the expression

$$L=\left[4\omega\left(\frac{\varepsilon}{\gamma_0}\right)^{1/2}\pm\frac{2\omega u}{T}\left(\frac{\alpha_T\varepsilon_T}{\gamma_0}\right)\right]^{-3/2}\left(\frac{\varepsilon}{\gamma_0}\right)^{5/4}, \tag{15}$$

where $f(x)$ and $f_T(x)$ are unknown functions to be determined, $x=kL$. Let us also introduce, according to [3], dimensionless parameters Γ_u and Γ_T:

$$\Gamma_u=\frac{1}{2}\left(\frac{\varepsilon}{\gamma_0}\right)^{-2/3}L^{4/3}\left(\frac{\partial u}{\partial y}\right)^2,$$
$$\Gamma_T=\frac{1}{2}\left(\frac{\varepsilon_T}{\alpha_T\gamma_0}\right)^{-1}\left(\frac{\varepsilon}{\gamma_0}\right)^{1/3}L^{4/3}\left(\frac{\partial T}{\partial y}\right)^2 \tag{16}$$

and designate the integrals that appear when transforming (11) and (12) with allowance for expressions (14):

$$\int_x^\infty x^{-2}f(x)\,dx=F^4,\quad \int_{x_0}^x x^2f(x)\,dx=\Phi^2,\quad \int_{x_0}^x x^2f_T(x)\,dx=\Phi_T^2. \tag{17}$$

We obtain from (11) and (12)

$$F^2[\Phi_1^2 + A_1\Phi_1 + B_1P] = 1, \quad F^2[P^2 + AP] = 1, \tag{18}$$

where

$$A = \delta\left[\left(\frac{\varepsilon}{\gamma_0}\right)^{1/3} \frac{\alpha_T \gamma_0}{\varepsilon_T} L^{4/3}\right]^{1/2};$$
$$A_1 = -4\omega\left[\left(\frac{\gamma_0}{\varepsilon}\right)^{2/3} L^{4/3}\right]^{1/2};$$
$$B_1 = \mp\frac{2\omega u}{T}\left[\left(\frac{\gamma_0}{\varepsilon}\right)^{5/3} \frac{\varepsilon_T}{\alpha_T \gamma_0} L^{4/3}\right]^{1/2}; \tag{19}$$
$$P^2 = \Gamma_T + \Phi_T^2, \quad \Phi_1^2 = 2(\Gamma_u + \Phi^2).$$

From the second of Eqs. (18) we have

$$P = \frac{-A + \sqrt{A^2 + 4/F^2}}{2}. \tag{20}$$

Having obtained the value of $\Phi_1{}^2$ from the first of Eqs. (18) with allowance for (20), and differentiating it with respect to x, remembering that

$$(\Gamma_u + \Phi^2)'_x = x^2 f(x), \quad F'_x = -\frac{f(x)}{4F^3x^2},$$

we determine the relationship between F and x in the form

$$x^4 = \frac{1}{4F^5}\left[1 - \frac{A}{\sqrt{A_1^2 - 4F_1}}\right]\left[\frac{1}{F} - \frac{B_1}{\sqrt{A^2F^2 + 4}}\right], \tag{21}$$

where $F_1 = B_1P - 1/F^2$. Differentiating (21) with respect to x, we obtain f(x):

$$f(x) = \frac{64F^9x^5}{20F^5x^4 + 1/F + f_1 - f_2 + f_3}, \tag{22}$$

where

$$f_1 = \frac{A_1B_1F^2F_3}{(A^2F^2 + 4)^{3/2}(A_1 - 4F_1)^{3/2}};$$
$$f_2 = \frac{A^2B_1F^2}{(A^2F^2 + 4)^{3/2}}, \quad f_3 = \frac{A_1(A_1^2 - 4F_1 - 2FF_2)}{F(A_1^2 - 4F_1)^{3/2}};$$
$$F_2 = \frac{2}{F^2}\left[\frac{1}{F} - \frac{B_1}{(A^2F^2 + 4)^{1/2}}\right];$$
$$F_3 = A_1^2A^2 - 4A^2F_1 - 2A^2FF_2 - 8F_2/F.$$

As is seen from (22), the function f(x), describing the spectrum of the turbulent energy, depends not only on x = kL, but also on the constants A, A_1, and B_1, defined by the adiabatic temperature gradient, rotational velocity, and values of ε and ε_T.

The quantities A_1 and B_1 are related by the scale L so that $A_1 + B_1 = \mp 1$.

By differentiating the second of Eqs. (18) with respect to x we obtain the function $f_T(x)$. Considering that $(\Phi_T{}^2)'_x = x^2 f_T(x)$, and with allowance for (20), we obtain

$$\frac{f_T(x)}{f(x)} = \frac{\sqrt{A^2F^2 + 4} - AF}{2F^6x^4\sqrt{A^2F^2 + 4}}. \tag{23}$$

Let us consider the asymptotic behavior of spectra in the interval of buoyancy, i.e., when $k \ll 1/L$ (small x and, hence, large values of F), and in the interval of inertial

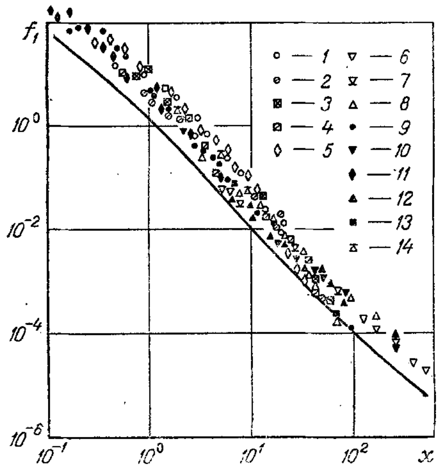

Fig. 1

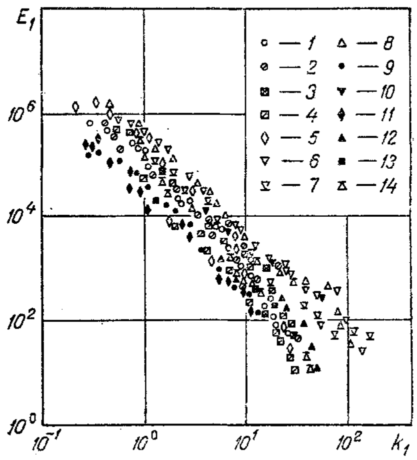

Fig. 2

Fig. 1. Graph of the model function f(x) and the experimental data. Solid line depicts the calculated relationship, the open points represent measurements near the outlet side of the blade; the filled points represent measurements near the inlet side; 5, 6, 7, 8, 9, 11, 14, measurements at the inlet section of the blade; 1, 2, 10, 13, at the middle part of the blade; 3, 4, 12, at the outlet of the blade; 1, 5, 8, 9, 13, at a distance y = 20 mm from the blade surface; 1, 2, 4, 6, 7, 10, 11, 12, 14, at a distance y = 0.5 mm; 3, 7, ω = 15.7 rad/sec; the remaining points, ω = 31.4 rad/sec.

Fig. 2. The measured values of the longitudinal spectrum of the velocity pulsation. The notation is the same as in Fig. 1.

convection at large x and small F, assuming for simplicity that the adiabatic temperature gradient is small and can be neglected.

It follows from Eqs. (21) and (22) that for stable stratification in the buoyancy interval, the falloff of the kinetic-energy spectrum obeys either "the 5/3 law" or "the 11/5 law," depending on the ratio between $B_1/2$ and $1/F$ in Eq. (21). For $1/F \ll B_1/2$ "the 11/5 law" will hold. The falloff of the temperature spectrum obeys either "the 5/3 law" or "the 7/5 law," respectively.

In the interval of inertial convection, both functions f(x) and $f_T(x)$ vary according to "the 5/3 law." For unsteady stratification, $x \to 0$ and $f(x) \to 0$ as $F \to 2/B_1$; since $f(x) \to 0$ also as $x \to \infty$ this means that the function f(x) has a maximum [3].

The formulas obtained hold for $x > x_0$, where x_0 is determined from Eq. (21) when $F = F_0$, and the value of F_0 is determined from the first or the second of Eqs. (18) for $\Phi^2 = \Phi_T^2 = 0$. For example, it follows from the second of Eqs. (18) that

$$F_0 = \frac{1}{\sqrt{\Gamma_T + A\Gamma_T^{1/2}}}.$$

If we neglect the heat transfer to the walls of the channel and also the adiabatic temperature gradient (the case that corresponds to gas flow in turbomachines with low rotational velocities), then f(x) is to be determined from a single Eq. (11), the solution of which is considerably simplified because the last term on the right-hand side of this equation vanishes.

Let us write the scale of buoyancy L, according to (15), as

$$L = (4\omega)^{-3/2}\left(\frac{\varepsilon}{\gamma_0}\right)^{1/2}. \tag{24}$$

Solution of Eq. (11) yields

$$x^4 = \frac{\varphi}{4F^6}, \quad f(x) = \frac{4\cdot 2^{1/2}F^{5/2}(F^2+4)}{3(F^2+4)+1/F-1}, \tag{25}$$

where

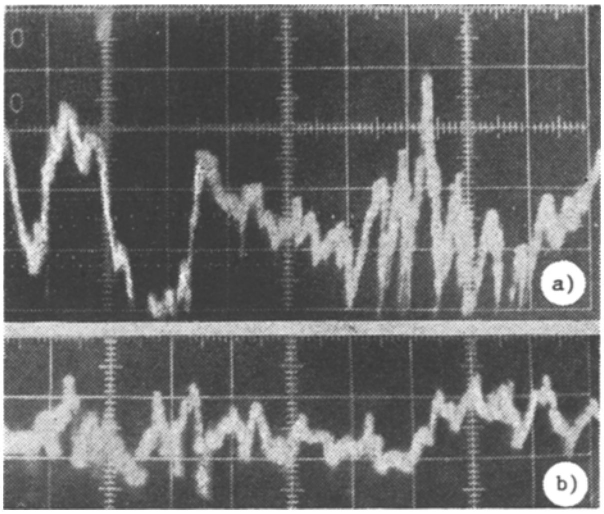

Fig. 3. Oscillograms of the pulsations of the longitudinal velocity: a) for the output side, at the middle of the blade, y = 0.5 mm; b) for the intake side, inlet section, y = 0.5 mm; amplification of oscillograph 5 mV/mm, scanning 0.5 msec/mm.

$$\varphi = 1 - \frac{F}{(F^2+4)^{1/2}}.$$

As is seen from (25), in the case under consideration f(x) is a completely universal function which depends solely upon x. Equations (25) hold for $x > x_0$, with the corresponding expression F_0 being determined from the first of Eqs. (18) for $A_1 = -1$ (on the basis of (24)); $B_1 = 0$, $\Phi^2 = 0$, $\Phi_1{}^2 = 2\Gamma_u$:

$$F_0 = \frac{1}{\sqrt{2\Gamma_u - (2\Gamma_u)^{1/2}}}. \tag{26}$$

Equation (25) shows that the asymptotic behavior of f(x) for both small and large values of x is described by "the 5/3 law"; however, the numerical coefficients for $x^{-5/3}$ are different.

In order to transform to the function $f_1(x)$ describing the one-dimensional spectrum, let us use the relationship [3]

$$f_1(x) = 1/4 \int_x^\infty \left(1 + \frac{x^2}{y^2}\right) \frac{f(y)}{y}\, dy, \tag{27}$$

which produces the following expressions for $f_1(x)$ for small and large x, respectively:

$$f_1(x) \sim \frac{4}{11} x^{-5/3}; \quad f_1(x) \sim \frac{13}{55} x^{-5/3}. \tag{28}$$

As is seen from (28), there should be an intermediate relationship for other values of x, such as is shown in Fig. 1; this dependence is well confirmed by the experimental data.

The turbulence was experimentally investigated in the channels of the wheel of a centrifugal compressor with the outer diameter of the wheel d_2 = 1000 mm, the width of channels at the outlet b_2 = 70 mm, and the rotational velocities ω = 15.7 and 31.4 rad/sec. The measurements were taken using sensors made of a tungsten wire of diameter equal to 5-6 μm and of a 2-mm base in conjunction with a T-7M hot-wire anemometer fabricated at Donetsk University. The root-mean-square (rms) velocity pulsations were measured by a V3-14 voltmeter, and the average velocity, by a VK-7A digital voltmeter. Velocity pulsations were recorded on magnetic tape, with subsequent decoding of the spectrum by a Model-2120 analyzer of the firm Bruel and Kjaer with a Model-2307 automatic recorder at a 10% frequency passband. The results were then processed using an "Odra" computer; pulsations were also photographed from the screen of an S1-19 oscillograph.

The signals from the sensors mounted on the wheel were transmitted to the devices with the help of a contact slip with copper brushes sliding on copper rings. The voltage drop in the brush contacts did not exceed 2-3 mV with rms signal levels from the sensors equal to 50-250 mV.

The results of measurements of spectra of pulsations of axial velocities at different distances from the channel sides are presented as the function $f_1(x)$ in Fig. 1 and as the function $E_1(k_1)$ in Fig. 2. The values of $E_1(k_1)$ were determined on the basis of the Taylor hypothesis of "frozen turbulence" from the equations

$$E_1(k_1) = \frac{wE_1(f)}{2\pi f}, \quad k_1 = \frac{2\pi f}{w}, \tag{29}$$

where $E_1(f)$ are the processed readings of the analyzer at a frequency f.

The velocity of dissipation ε, required for calculating the scale L, was determined by integrating the experimental relationship according to the equation

$$\varepsilon = 15\nu \int_{k_1}^{k} k^2 E_1(k)\, dk. \tag{30}$$

It is seen from a comparison of Figs. 1 and 2 that graphing the experimental data in the $f_1(x)-x$ coordinates allows us to cluster them closely around the model curve plotted according to (25); for clarity the curve is shifted upward by the value $\Delta \log f_1(x) = 0.5$. In Fig. 3, oscillograms of the pulsations of the longitudinal velocity at the outlet and inlet sides of the channel are shown. As is seen from Figs. 2 and 3, the density level of the kinetic energy of the turbulence and pulsations of the velocity is higher at the outlet side of the channel, where amplification of the turbulence and, according to [2], an increase in the dynamic velocity are observed.

NOTATION

A, B, constants; E(k), $E_T(k)$, $E_{TV}(k)$, spectra of kinetic energy, temperature, and mutual spectrum of temperature and velocity fields; K, turbulent viscosity coefficient; L, buoyancy scale; T, temperature; X, volume forces; c_p, heat capacity at constant pressure; f, frequency; g, acceleration of gravity; k, wave number; x, y, z, coordinates; u, v, components of the mean velocity; α_T, ratio of the turbulent-viscosity coefficient to the heat-transfer coefficient; γ_0, constant; δ, adiabatic temperature gradient; ε, ε_T, energy dissipation velocity of the turbulent flow and levelling of the measure of the thermal field heterogeneities; ν, kinematic-viscosity coefficient; χ, thermal diffusivity; ω, angular velocity of rotation.

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